DOCUMENT RESUME

ED 399 262 SP 036 932

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TITLE Developing Mathematics Reform: What Don't We Know about Teacher Learning--But Would Make Good Working Hypotheses? NCRTL Craft Paper 95-4.
INSTITUTION National Center for Research on Teacher Learning, East Lansing, MI.
SPONS AGENCY Office of Educational Research and Improvement (ED), Washington, DC.
PUB DATE Oct 95
NOTE 53p.; Paper presented at a conference on Teacher Enhancement in Mathematics K-6 (Arlington, VA, November 18-20, 1994).
AVAILABLE FROM National Center for Research on Teacher Learning, 116 Erickson Hall, Michigan State University, East Lansing, MI 48824-1034.
PUB TYPE Reports - Research/Technical (143) -- Speeches/Conference Papers (150)

EDRS PRICE MF01/PC03 Plus Postage.
DESCRIPTORS *Educational Change; *Educational Policy; Elementary School Mathematics; Elementary Secondary Education; *Faculty Development; *Knowledge Base for Teaching; *Mathematics Instruction; Mathematics Teachers; Secondary School Mathematics; *Teacher Improvement

ABSTRACT

This paper examines what teacher educators, policymakers, and teachers think they know about the current mathematics reforms and what it takes to help teachers engage with these reforms. The analysis is organized around three issues: (1) the "it" envisioned by the reforms; (2) what teachers (and others) bring to learning "it"; (3) what is known and believed about teacher learning. The second part of the paper deals with what is not known about the reforms and how to help a larger number of teachers engage productively with the reforms. This section of the paper confronts the current pressures to "scale up" reform efforts to "reach" more teachers. Arguing that what is known is not sufficient to meet the demand for scaling up, three potential hypotheses about teacher learning and professional development are proposed that might serve to meet the demand to offer more teachers opportunities for learning. (Contains 81 references.) (Author/ND)

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Craft Paper 95-4

Developing Mathematics Reform: What *Don't* We Know About Teacher Learning -- But Would Make Good Working Hypotheses?

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SB036932

Sponsored by the United States Department of Education
Office of Education Research and Improvement

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Published by

National Center for Research on Teacher Learning
116 Erickson Hall
Michigan State University
East Lansing, Michigan 48824-1034

October 1995

This paper was invited for a conference on Teacher Enhancement in Mathematics K-6, in Arlington, Virginia, November 18-20, 1994. The ideas and opinions expressed herein are mine, and do not represent the official position of Michigan State University or of the U.S. Department of Education, Office of Educational Research and Improvement.

This work is sponsored in part by the National Center for Research on Teacher Learning, College of Education, Michigan State University. The National Center for Research on Teacher Learning is funded primarily by the Office of Educational Research and Improvement, United States Department of Education. The opinions expressed in this chapter do not necessarily represent the position, policy or endorsement of the Office or the Department.

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National Center for Research on Teacher Learning

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[1]Formerly known as the National Center for Research on Teacher Education (1985-1990), the Center was renamed in 1991.

Abstract

This paper begins with an examination of what teacher educators, policymakers, and teachers think they know about the current mathematics reforms and about what it takes to help teachers engage with these reforms. The analysis is organized around three questions:

1. What is the "it" envisioned by the reforms?

2. What do teachers (and others) bring to learning "it"?

3. What do we know and believe about teacher learning?

In the second part of this paper, I pose the question, "What is it that we don't know about the reforms and how to help a larger number of teachers engage productively with them?" This section of the paper confronts the current pressures to "scale up" reform efforts to "reach" more teachers. Arguing that what we "know" is not sufficient to meet the demand for scaling up, I propose three potential hypotheses about teacher learning and professional development that might serve to meet the demand to offer more teachers opportunities for learning.

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Paper prepared for conference on
Teacher Enhancement in Mathematics K - 6
Arlington, Virginia
November 18 - 20, 1994

Deborah Loewenberg Ball[2]
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Preface

My task in this paper is to try to frame a set of issues around teacher learning in relation to teaching mathematics. The problem is to take stock of what we know in ways that might help us respond to the Congressional mandate to "reach" many more teachers as they "implement" reform.[3] But the NCTM *Standards* documents are far from a program to be implemented.[4] Rather, they are two sets of inspirational, but unfamiliar and incomplete visions for improving mathematics education. Because they are underdetermined, I propose to change the question slightly. Rather than asking, "How might we help more teachers implement the reform?" I want to ask, "How might we engage a wider community in

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[2]I am indebted to several of my colleagues for their careful readings and comments on various drafts of this paper: Suzanne Wilson, David Cohen, Magdalene Lampert, Angie Eshelman, Robert Floden, Ruth Heaton, Sue Poppink, Kara Suzuka, Dirck Roosevelt, Elizabeth Fennema, and Deborah Schifter. I also wish to gratefully acknowledge Angie Eshelman for her assistance with the paper.

[3]In 1993 the U.S. Senate Appropriations Committee mandated that the Department of Education and the National Science Foundation were to "increase teacher training activities" between 1993 and 1998 to provide "intensive pedagogical and disciplinary training" in mathematics and science to 600,000 elementary teachers. While "scaling up" the numbers of teachers "reached," the funding agencies were also to ensure that the professional development provided was systemic, high quality, research-based, and consistent with the reforms.

[4]See Ball, 1992.

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developing and enacting reform?" Although these questions are different, teacher learning is at the heart of both of them. Yet selecting the latter over the former has significant implications for what I discuss below, as well as for moving the reforms forward.

I begin with a few assumptions. First, although reforming mathematics is inescapably dependent on political leverage, economic resources, and public marketing,[5] at its heart is a problem of learning.[6] Yet the idea that reform requires learning is often too quickly agreed to, and inadequately examined. And so I begin with a short analysis of why the mathematics reforms require learning, and the nature of that learning. Second, I assume that there are some things we know about teacher learning. And that it might be important to consider our current "knowledge base" in this domain. Finally, I assume that such "knowledge" is a conglomerate of belief, wisdom of practice, folklore, myth, and constraint, and that it would be useful to identify and examine some of what we really may *not* know, for it might well be that the seeds of fruitful work lie in what we *don't* know.

I begin this piece by exploring the question, "What is it that we know about the math reforms and what it takes to help teachers engage with them?" I examine this in three sections:

1. What is the "it" envisioned by the reforms?
2. What do teachers (and others) bring to learning "it"?

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[5]E.g., Mirel, 1994.

[6]See Cohen & Barnes, 1992. The notion that policymaking and reform are centrally matters of teaching and learning is one that has been at the heart of our work on the Education Policy and Practice Study at Michigan State University and the University of Michigan. For this theoretical perspective, as well as many other ideas about reform, I would like to acknowledge David Cohen, Suzanne Wilson, and Penelope Peterson, my co-principal investigators in this research.

3. What do we know and believe about teacher learning?

In the second part of this paper, I pose the question, "What is it that we *don't* know about the reforms and how to help a larger number of teachers engage productively with them?" With a reexamination of some prematurely-dismissed paths of professional development, I explore some potential working hypotheses about helping people learn to do "it."

What We Know

In search of "this kind of teaching": What is the "it" envisioned by the reforms?

The multiple foundations of the reform. These are times of ambitious efforts to reform mathematics curriculum and instruction. A host of prominent national reform documents paint a vision of challenging mathematics instruction for all students.[7] The reform's rhetoric takes aim at both patterns of inequity and curricular inadequacy. Students are to learn mathematics with understanding, engage in and be able to solve real-world and meaningful problems, and develop the confidence and power to think mathematically. And these goals are for all students, with particular concern for those who have been traditionally underserved and excluded from mathematics: students of color, poor students, girls.

The foundations for this reform are complex. As part of a tidal wave of education reform, the mathematics education reforms are based on both dissatisfactions with what is, and new aspirations for what could be. On one hand, critics worry about U.S. achievement in

[7]E.g., National Council of Teachers of Mathematics, 1989, 1991; National Research Council, 1989; 1993 (a and b).

mathematics. As a society, we seem to be educating only a tiny fraction of our population to be mathematically literate. For reasons economic, political, and social, the coming decades will demand many more people who can use mathematics competently. Critics also point to the sociological patterns in participation and achievement that show dramatic mathematics dropout rates among female, minority, and poor students.

On the other hand, the reforms also grow from aspirations for what could be. New ideas about learning and knowledge have led to revised Deweyian images in which students interact around important questions as members of learning communities, developing knowledge together. Constructivist theories of learning[8] have deeply permeated contemporary theoretical work in mathematics education. At the same time, knowledge itself is seen as less definite and more situated in the assumptions and agreed-upon ways of working shared by a community. There is more attention to questions of mathematical argument and discourse -- methods of proof, tools of representation. Although there is far from consensus on "the" nature of mathematics, fallibilist and quasi-empirical views of the discipline have gained increased attention.[9]

Even this brief sketch highlights one central challenge: the scope of the learning that such reforms would demand, not just for teachers but for everyone. The patterns of poor achievement are embedded in the structures of school, inherited ideas about curriculum and

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[8]See Cobb, 1994, for an excellent treatment of the interaction of constructivist, social constructivist, and sociocultural ideas about learning.

[9]E.g., Lakatos, 1974

about who can learn, and shared images of teaching. These would have to change. And commonsense theories of knowledge as fact, and of learning as remembering information are at odds with these new ideas.[10] These theories too would have to be shaken at their roots. Clearly, reforming mathematics is no short order: It would require profound and extensive societal and individual learning -- and unlearning.

But the problem is more complex because of a special quality of the reforms: Despite the laudable rhetoric, what is specifically implied for classroom practice is far from definite. This is our first challenge: that the "it" to which we aim is no clear program for practice.

Inspiring visions, uncertain practices. The NCTM *Standards*,[11] widely-touted for their vision, offer perhaps the most detailed images. With vignettes, examples, and illustrations, the *Standards* books -- 454 pages' worth (with a third volume on assessment to follow soon) -- are one main resource for reformers. Despite their persuasive, inspiring vignettes, however, these documents are far from programs for practice. They sketch directions and commitments, principles and aspirations. They cannot provide guidance for the specifics of minute-to-minute practice or for the decisions met day to day. For example, one of the teaching standards, in envisioning the teacher's role in classroom discourse, states that teachers have to decide "when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty."[12] True enough. But the challenge is to judge when to do

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[10]In his 1989 essay, David Cohen argues persuasively and elegantly that efforts at "ambitious teaching" run repeatedly up against prevailing societal views of knowledge as factual, unchanging, and certain.

[11]National Council of Teachers of Mathematics, 1989, 1991.

[12]NCTM, 1991, p. 35.

which, and on what basis. When, for instance, is a disagreement among students something worth continuing? When should the teacher step in and clear up controversy? When is a particular student's statement best left alone? When is it good to probe?[13] The *Standards* also speak of "worthwhile mathematical tasks" and specify some elements of such tasks. But, with a particular group of students, what makes a task likely to be productive of learning is much less straightforward -- and helping it to be so, even less clear. Sometimes good tasks fizzle to nothing, or run into unanticipated difficulties. How, specifically, can a "good" task be best framed and orchestrated with a particular class?

Some might argue that these questions cannot yet be answered because the reforms are too new and, thus, underspecified. The air is filled with words about which there has been little discussion -- problem solving, understanding, meaningfulness, autonomy, authenticity, inquiry. Some assume that, with time, the specifics will be worked out: We will know more and will therefore be able to develop more explicit and helpful guidance for teachers.

Explicating the vision more fully is certainly an important challenge of the reforms. And it will help to have more, and better-specified, articulations of the ideas and their interpretations. Yet no matter how much more specific the vision becomes, it will not be close to a prescription for practice. Shulman (1983) argues that initiatives for change can at best be "a shell within which the kernel of professional judgment and decision making can function comfortably." He argues that such initiatives *cannot* determine directly teachers' actions or decisions, and he concludes that they can, at best, "profess a prevailing view, orienting

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[13]See Chazan & Ball, 1995.

individuals and institutions toward collectively valued goals, without necessarily mandating specific sets of procedures to which teachers must be accountable."[14]

This relationship between policy and practice is accentuated in the case of the current mathematics reforms. With an eye on new goals of "understanding," teachers ply their trade still closer to the uncertainties of learning and knowledge. This new vision of teaching confronts -- and embraces -- the uncertainties of learning and teaching, and the interaction between the two.[15] Three sources of uncertainty stand out as endemic to this kind of teaching: the inherently incomplete nature of knowledge in teaching; the multiple commitments with which teachers work; and trying to teach in ways that are responsive to students.

Uncertainty is not a comfortable idea, and certainly not a happy prospect for ambitious reformers. What does it mean? To begin with, knowledge in teaching is most often incomplete. Human understanding is far from a simple, visible phenomenon. To illustrate, I use an example from my own teaching of third grade.[16] One day, in the midst of several weeks of work on fractions, my third graders drew pictures of 4/4 and 5/5 in their notebooks.[17] Each picture looked like this:

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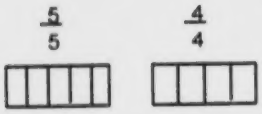
[14]Shulman, 1983, p. 501.

[15]E.g., Cohen, 1989, in preparation; Jackson, 1986; Lampert, 1985.

[16]As part of my research, I teach elementary school mathematics on a daily basis. The episode I relate here occurred in my third grade class during the 1989-90 school year. The data on which I am drawing were gathered as part of an NSF project which involved documenting the mathematics teaching and learning across the entire school year in my class and in the fifth grade class of my colleague, Magdalene Lampert. The teacher in whose classroom I was working was Sylvia Rundquist (see Ball & Rundquist, 1992, for a description and examination of our four-year collaboration).

[17]This example is discussed in Ball & Wilson, in press.

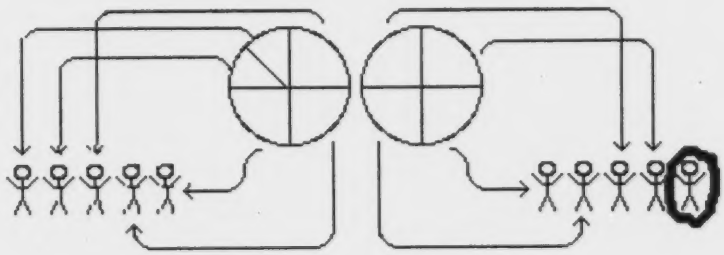


Had I stopped there, I would have concluded that all my students knew that 4/4 was equivalent to 5/5 (even if they would not have used the term "equivalent"). When I asked which was more, 4/4 or 5/5, about half the students thought 5/5 was more and some thought they were the same. One child thought 4/4 was more than 5/5. Why? How could they look at these pictures and think that? As we talked, I discovered that some students thought that since 5/5 has "more pieces," it was actually more. Even though they gave the "correct answer," it was not clear what the students who said the two quantities were "the same" were thinking. At the end of class, I asked everyone to write in their notebooks about 4/4 and 5/5. As I studied what they wrote and drew, I could not tell for sure what they understood. But what they did say and draw made me wonder, make conjectures, and proceed to plan out the next day using my hypotheses. On an ongoing basis, teachers are faced with making such judgments, designing next steps, evaluating students' learning -- and with incomplete and indefinitely interpretable evidence.

No matter what kind of research we do in the future -- exploring students' knowledge and preconceptions, examining what they know and how -- teachers will continue to confront such uncertainty on a daily basis. Can a teacher become more skillful at probing and making sense of students' ideas? Yes. But what teachers know about their students can never be certain or complete.

The practice of teaching itself is uncertain as well. Teachers work in the midst of multiple and oft-competing commitments.[18] For example, at the core of the reform visions is the commitment to teach worthwhile content with intellectual integrity. But equally at the core is the commitment to honor students' ideas. When a child presents a novel approach to a problem that is imaginative -- and completely nonstandard -- what is the right thing for the teacher to do?

This is seldom an easy question to answer. In the episode above, Sheena, one of the African-American girls in my class, argued articulately that 5/5 had to be more than 4/4. She went to the board and presented her original (and persuasive) explanation, rooted cleverly in assumptions about sharing cookies. She drew two circular cookies, dividing them into four and five pieces, and showing that with 5/5 there is enough to pass out one piece to each of your five friends but with 4/4 one friend will not get any cookie.



It was important to me that Sheena -- a student of color, a quiet girl -- displayed enough confidence in herself and her ideas to defend them in the face of classmates' objections. And

[18]Goodlad, 1984; Lampert, 1985.

she is right, given the question she has framed ("Which way of cutting the cookie -- into fourths or fifths -- will serve more friends?"). Her drawing is another source of uncertainty. Most adults to whom I have shown this picture immediately assume that Sheena does not know that fractions must have equal pieces. But this is actually not clear. Dividing circles into fifths is technically complicated (try it!) and we had not done this in class. I knew that with other pictures, the children had sometimes said, "I know my picture isn't quite right, but just assume that the pieces are the same size." Although Sheena did not say that here, I am not so quick to conclude what she knows -- or does not know -- about equal parts.

As I listened to Sheena, I knew that next year's teacher might not be charmed by Sheena's way of thinking about this. She might see Sheena as lacking mathematical skills. *Was* she? Sheena *could* complete standard fraction worksheet items correctly (e.g., shade 3/4 of a rectangle) and she got the fraction items right on the end-of-year standardized test. Yet this nonstandard part of Sheena's thinking made me wonder. And I was aware that my twin commitments to teach mathematics with integrity and to honor students' ideas and ways of thinking were in tension in this case: Sheena was being creative. And some aspects of her answer were "right." But her nonstandard approach had actually changed the question. And her response to the original question was "wrong." What should be the "right" answer for me here? To this day, that remains uncertain. The slogans "teaching for understanding" and "mathematics for all" are a lot more complex when viewed up close.[19] Teaching often sits uncomfortably in the cross-talk of several such worthy -- and competing -- commitments.

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[19]Theule-Lubienski, in preparation.

Wrestling with these in context, on an ongoing basis, is a second source of the uncertainty of teaching.

Third, the kind of teaching envisioned by the reforms aims to be responsive to students, to what they say and do. Teachers often have to adapt and improvise in the face of what happens as lessons unfold. When my students drew the pictures of 4/4 and 5/5 correctly, but then believed that these were still not "the same amount," I had to remap where we were and where we might go. I realized that the phrase "the same amount" was fragile, and searched my mind for new phrasing. I noticed the ambiguity of the idea of "more" -- and adaptively began to consider another way to confront the problem that would allow us to explore equivalence without burying the students' alternative interpretations.[20]

Teaching is an interactive practice in a messy terrain of content, politics, and social and individual improvement.[21] Teachers work with and "on" other human beings, and such work is interdependent in ways that make it quite clearly different from the practice of mathematics or even biology where practitioners (biologists) work with and on materials which they cannot entirely control. This casts a light on the challenge of reform that makes it still more challenging and uncertain.

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[20]What I mean by this is that I wanted a way of working with the idea of "equivalence" that would facilitate their learning without simply covering up their ways of thinking with a convincing structure that would simply elicit the right answer. I discuss this in more detail in Ball & Wilson (in press).

[21]My thinking on this issue has been substantially informed and influenced by David Cohen's writing on teaching as a practice of human improvement, and the complexities that this presents for practice itself (Cohen, 1989, in preparation).

My sketch here is intended to illustrate one central issue, an issue too often bypassed in our discussions of teacher development in the context of these new reforms: The reforms do not prescribe a specific and identifiable practice. There exists no single "it" to which the reforms aim. Rooted theoretically in the theories and commitments sketched above, the NCTM documents -- as well as other reform documents -- are strong on promise, weak on existence proofs.[22] Despite their inspiration in progressive educational ideas, no one has fully developed these ideas on any scale in public schools or even in educational scholarship -- at least not to the level of practice. Considerable work lies ahead if the ideas of the reforms are to permeate practices in school in ways that are consistent with their intent. Such work would involve turning these dreams into conjectures, testing them out, revising the revisions.

What does developing "it" require? Although considerable debate exists about what the "stuff" of teaching is (and what counts as knowledge of teaching is perhaps more contested among teachers than mathematical knowledge is among mathematicians), there are nonetheless ideas, principles, insights, theories, and ways of doing things that practitioners know and use. For example, practiced elementary teachers are always on the lookout for opportunities for meaningful counting and can engage young children in enumerating, comparing, sharing equally. Many teachers know that manipulatives can be helpful in helping children to develop mathematical concepts. Most elementary teachers have a repertoire of ways of gaining the attention of the group, and most know particular problems that consistently interest fifth

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[22]Tyack & Tobin's (1994) notion of a goal as a *hypothesis* appeals to me, for it is neither naively overdetermined -- goals as "fixed targets" -- nor capriciously unfounded -- goals as mere wishes.

graders. This "stuff" is specialized knowledge of the domain, for it includes a host of things that the ordinary adult, even a parent, does not necessarily know. Note that this body of knowledge is both propositional and procedural -- for example, topics that interest students, how to read aloud, ways to gain a class's attention.

In addition to such pedagogical knowledge, there is knowledge of mathematics and knowledge about students. Much has been written in the past decade about the nature of the mathematical understandings that are crucial to teaching mathematics for understanding.[23] And, equally important, are understandings of the diversity of students with whom teachers work.[24]

Teaching also involves considerable skill -- such as listening to one child while watching thirty others, using one's voice as a tool, "reading" and interpreting the reactions and understandings of others who may communicate differently from the teacher, keeping a wide range of details in mind, posing appropriate questions. The kinds of things that play a role in practicing as a teacher of groups of children is more complex than revealed by our usual lists of what teachers need to know. A host of personal qualities matter: patience, curiosity, generosity in listening to and caring about other human beings, confidence, trust, and imagination.[25] There is caring about seeing the world from another's perspective, as well as enjoying the humor, sympathizing with the confusion, and caring about the frustration and

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[23]E.g., Ball, 1991; Russell, Schifter, Bastable, Yaffee, Lester, & Cohen, 1994; Simon, 1993; Wilson, Shulman, & Richert, 1987.

[24]E.g., Anderson, 1989; Carpenter & Fennema, 1992; Grant & Secada, 1990.

[25]See Roosevelt, 1994.

shame of others. And there are things like tolerance for uncertainty, willingness to take risks, and patience with confusion and mess. The personal resources which teaching demands are not so often discussed, and even less often nurtured. Is the kind of patience that teaching requires something that can be learned? Can empathy grow? If these kinds of resources and qualities are central to teaching, then we need ways of thinking about what might be ways of cultivating and nurturing their development.

But knowledge and skill are not all there is to learning teaching. Another crucial dimension centers on learning to reason and to construct new knowledge in teaching. Three factors underlie this imperative: one rooted in the demands of learning to teach, one in the particularities of practice, and one in the nature of knowledge. As a matter of individual development, learning to teach takes time. It requires taking ideas and images, skills and commitments, and developing a repertoire of professional practice with them. Teaching is also context-specific. Even skilled teachers must adapt their practice in particular situations. Finally, knowledge in teaching is both incomplete and contested, teachers are continually in the position of interpreting conflicting evidence and making choices and judgments. Because knowledge is incomplete, teachers must figure out new things as they teach. They are constantly faced with the data of their own experience. They must develop knowledge of particular children, of the material they are teaching, and of ways to engage students in the content. And, because knowledge is contested, teachers must have ways of working through the alternatives they are offered. They must somehow take stock and assess the relative merits of alternative ideas, interpretations, and strategies. If teachers are to be able to do anything

but respond entirely randomly to the flood of claims that are made about what works, what is true, and how things should be done, then they must be able to identify and weigh justifications for various claims.

Confronting discontinuities and discomforts of role, content, and practice: What do teachers (and others) bring to learning "it"?

There is a gradual recognition that teachers, just like their students, bring experiences and prior understandings that shape their learning.[26,27] These previous experiences often do not help them as they struggle to enact these new reforms. At times, past experiences can act as obstacles. For example, elementary teachers, most of whom experienced school knowledge as given -- and who acquired facts and memorized rules -- must invent a teaching that engages students in complex reasoning in authentic tasks and contexts. They are faced with trying to find ways to connect students with mathematics and mathematical reasoning, to engage students in genuine experiments. Even though schools have never taught all students equally well, teachers are to find ways to help all of their students.

And so a paradox emerges: Elementary teachers are themselves the products of the very system they are now trying to reform. An overwhelming proportion are women, and the majority did not pursue mathematics beyond what was minimally required. Many report their own feelings of inadequacy and incompetence, and can even recall experiences which became

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[26]E.g., Ball, 1988; Borko, Eisenhart, Brown, Underhill, Jones, & Agard, 1992; Brown & Borko, 1992; Schifter, 1993; Simon, 1993.

[27]Although I focus here on teachers, these comments could be made just as well about any of the many people who have a role in mathematics reform -- teacher educators, parents, school administrators, etc.

turning points when they decided to stop taking mathematics. Rather than becoming critical of the way we "school" mathematics, they often assume that their experiences are due to their own mathematical lacks and to the inherently useless content of mathematics. Those same experiences have equipped them with ideas about the teacher's role, about who can learn mathematics, and about what it takes to learn and know mathematics. Moreover, what teachers bring is not purely cognitive, for some of what they bring includes commitments about how to act with different students, a sense of themselves as helpful and effective, values about a kind of classroom environment. These, too, influence their interpretation of and disposition toward the mathematics reforms.

This mix of things that teachers bring become evident in concrete contexts -- such as in viewing videotape or discussing a case. It becomes clear that, given what people's own past experiences are, the reform visions are simultaneously appealing and unsettling, attractive and unfamiliar. When people view and discuss videotapes of alternative approaches to mathematics teaching, they have mixed reactions.[28] On one hand, they are impressed with the children's confidence and civility. They are attracted by the students' flexible use of drawings and analogies, as well as their articulateness. On the other hand, viewers also find it deeply disturbing to hear the array of students' interpretations. Evidence that students may not

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[28]I am grateful here to the many teachers and teacher educators who have shared with me their experiences with videotapes from my third grade class, as well as to the people with whom I have viewed and discussed classroom tapes.

understand is not always intriguing, for it can be quite uncomfortable.[29] One major source of teachers' sense of efficacy and satisfaction is the sense that they can help students learn.[30] And when we do not ask students to voice their ideas, we run less risk of finding out what they do and do not know. In asking students to talk and otherwise represent publicly their thinking, the gap between their thinking and ours becomes visible. And the ensuing instinct to explain away the apparent misunderstanding is strong: "Did the teacher use manipulatives to show this?" "Had the students been told that the unit has to be the same?" Deeply rooted in teachers is the impulse to help and clarify, to show and tell. It is a good and worthy instinct -- and quite right. Teachers are, after all, responsible for helping their students learn. Old complacencies about understanding are called into question when one starts listening more closely to students. My students who drew rectangles representing 4/4 and 5/5 the same seemed to understand equivalence until I began asking more questions. Their earlier correct worksheets notwithstanding, I began to see a glimpse of understandings less robust than I had hoped. Moving in the direction of the mathematics reforms means confronting the uncertainties, ambiguities and complexities of what "understanding" and "learning" might really mean and entail.

But things are more complicated still. If student understanding becomes more problematic, one's own understandings are soon more uncertain as well. And this is at least as

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[29]Of course, students can also display exquisite understandings of very complex ideas, glimpses of which are breathtaking. I concentrate deliberately on the less rosy side of listening more closely to students' thinking, for I think it has important implications for what it means to change one's teaching in the direction of the mathematics reforms.

[30]Lortie, 1975; Smith, 1994.

unsettling. After all, teachers are "supposed to" know what they are teaching. Confronting one's own uncertainties in understanding can make a teacher feel inadequate and ashamed.[31] That the mathematics reforms are aimed at helping students understand content in usable and powerful ways is part of the appeal for teachers whose own mathematical histories did not offer them such opportunities. Still, in pursuing such goals, deep anxieties about one's effectiveness and knowledge are likely to surface.

Encounters with the reform visions can be deeply uncomfortable. Despite the obvious fascination of children's nonstandard thinking, if the goal is to help students master content, close views of students' alternative interpretations can threaten established practices. If teachers do the things they have always assumed were helpful and then find that students are thinking differently than what they hoped, this is still more troubling.[32]

The mathematics reforms are attractive and inspiring in many ways. Yet there are also reasonable and powerful disincentives to engage with this agenda, some of which are deeply personal and at the heart of who one is in trying to be a good teacher. Often teachers must defend to parents and administrators things they are trying even before they themselves are

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[31]Several poignant and profound examples of this exist in the literature. Heaton (1994) writes about her own struggles to delve into what she had previously considered simple mathematical ideas. Rundquist (Ball & Rundquist, 1992) also describes how vulnerable it made her feel to discover her own confusions, and how personally risky it was to write about it. Other examples can be found in Schifter's (in press a and b) books, particularly the chapters by Toney and Yaffee.

[32]Dick Prawat (personal communication) studied a teacher, who, as she began to decouple learning as a direct outcome of sincere and imaginative teaching, began to doubt that she had ever helped her students "really" understand. As a dedicated twenty-year veteran, she was profoundly distressed.

convinced or confident. A risky prospect at best, being in this position is understandably unappealing.

Some of what makes learning this kind of practice especially hard includes feeling disconnected from one's past experience with schools and from practice in many contemporary classrooms. With the commitment to attend to what teachers bring, we need to be sensitive to the loss that making a commitment to mathematics reform can engender. What one remembers from third grade is much more useful when trying to learn a more conventional version of practice than it is when one is learning a more novel version of practice. Never having explored the territory this way as a mathematics learner can be disorienting. One's own mathematical understanding and one's ability to listen for and interpret students' thinking plays an important orienting role in navigating the territory in new ways. Being a reformer, an agent for change, can be hard. It takes courage and risk-taking. It takes being adventurous and willing to experiment and try new things in a context that has not typically rewarded or encouraged innovation or rebelliousness. Support, in various forms of communication and community with colleagues engaged in this work, is needed in a very real way.

What do we know and believe about teacher learning?

What we know and believe about teacher learning is beginning to be codified in a growing literature. It is also reflected in practice, in the patterns and innovations of teacher change and support.[33] Perhaps, most of all, it is reflected in our talk and the assumptions we

[33]Just as teachers' knowledge can be examined in the contexts of their work -- their evolving wisdom of practice -- so, too, is knowledge about teacher learning evident in the practice of professional development.

reveal about professional development. With a panorama of the literature and programs, I offer below for our collective consideration a tentative list of widely held beliefs about teacher learning.[34] It should be understood that the status of these as "knowledge" is problematic, for the empirical bases for these beliefs vary widely. Some have been investigated in studies of teacher learning and teacher education while some represent current dominant ideology. Even some of those supported by research are the product of studies conducted by teacher educators who design a teacher education experience rooted in one or more of these beliefs. Promising results are then used as evidence for the original assumption. The items on this list, then, possess uneven warrants. They also focus on some aspects of teaching and teacher learning, and not others. But, ubiquitous, they are widely promoted.

1. What teachers bring to learning to teach -- prior belief and experience --affects what they learn. Increasingly, teachers' own histories -- personal and professional -- are thought to play an important role in what they learn from professional development experiences.

2. Learning to create the kinds of teaching envisioned by the mathematics reforms takes a long time and is hard.[35] Changes do not happen overnight, nor simply by deciding to teach differently. There is as much to unlearn as there is to learn, and what there is to learn is complex and underdeveloped. In ways not well-understood, the odyssey probably entails (at some level) revising deeply-held notions about learning and knowledge, reconsidering one's assumptions about students and images of oneself both as mathematical thinker, a cultural

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[34]Because my intention here was to propose a short list of shared ideas, I did not attempt the ambitious task here of citing all the studies and programs that have contributed to this list. I drew widely on articles in the field, my experience reviewing NSF, NCTM, and AERA proposals, chapters in the just-published NCTM 1994 Yearbook on *Professional Development for Teachers of Mathematics*, as well as the program descriptions for this conference on professional development for mathematics reform.

[35]Although acknowledged widely, what actually makes it hard does not seem to be well-understood nor finely-articulated.

and political being, *and* teacher[36] -- all this, and developing new ways of teaching, reflecting and assessing one's work.

3. Often, the most effective staff development model involves follow-up, usually in the form of long-term support (e.g., opportunities to meet with others engaged in the same process) and coaching in teachers' classrooms. Other means that could help teachers continue to develop and learn might also fit this notion of "follow-up."

4. Teacher educators and staff developers should model the approaches which they are promoting. This is an oft-heard maxim, quite variously interpreted.

5. Subject matter knowledge matters in learning to teach for understanding. Selecting a generative problem or task for students requires being able to "see" the mathematics latent in its scope. And trying to use tasks and problems -- in ways that exploit their potential and support student learning -- depends on the teacher's own mathematical understandings. To orchestrate a class discussion of a mathematical conjecture can be treacherous when the teacher feels unsure of the terrain being explored. The teacher's own mathematical knowledge is also an important resource in interpreting students' unexpected statements and solutions.

6. Knowledge of children and their mathematics is crucial to teaching for understanding. Learning more about students, and about listening to them is crucial. How to *hear* what students say is more than a matter of acuity, for it requires seeing the world through another's eyes and perspective, not at all an easy task (especially when those worlds are diverse, sometimes disparate).

7. The contexts in which teachers work affect what they can do. (Included in "context" are students, parents, administrators, tests, district and state objectives and curricular guidelines.) Most often discussed are the ways in which aspects of the context constrain and inhibit teachers' efforts. Students unfamiliar with this kind of teaching resist. Parents protest departures from customary practice. Administrators are intolerant of less-orderly classrooms or fail to provide teachers with materials or time to develop their practice. External curricular guidelines mandate pacing and coverage and impede teachers who want to teach for understanding. There is less understood about promising extant resources although many have claimed that the community can be a significant resource in

[36]E.g., Toney, in press; Weissglass, 1994.

making reform, that new curriculum and assessments can serve as levers for reform.

8. Reflection is central to learning to teach. For the most part, this perspective focuses on structure and context, emphasizing that teachers need time, space, and encouragement to reflect in ways that facilitate their learning -- by talking with others, by keeping a journal, by engaging in action research. Less attention is paid to the specific objects, contexts, and nature of what teachers might reflect *on* and *with*, leaving somewhat out of focus questions about the variety of learnings this might support.

9. Teacher development is especially productive when teachers are in charge of the agenda, determining the focus, nature, and kind of programming or opportunities. In the name of professional autonomy, many argue that teachers should determine the shape and course of their own development. Little discussion emerges about the dilemma this raises in working toward reform. When teachers set the agenda for their own professional learning, they are likely to be limited by their current vistas. Setting oneself off into a terrain beyond one's current horizons is difficult, if not impossible. Yet, when others set the agenda, they are not necessarily more likely to have vision or sensitivity to teachers' needs and concerns. How to design provocative experiences for teacher learning, that hold real potential for change, for engagement with what is hard about the reforms, and yet that also honor teachers as professionals, is a matter more complex than this maxim suggests.

WHAT WE DON'T KNOW

Although these beliefs are widely shared, they are far from a majority view when one considers the enormous "staff development industry." Districts, counties, and private entrepreneurs sponsor workshops, institutes, and after-school dinner meetings to develop, train, refresh, update, and inservice teachers.[37] Administrators form committees, bring in experts, adopt new textbook series. Teachers read *Teaching Mathematics, Instructor, Learning*, and *American Educator*. They purchase commercial black-line masters for

[37]Sparks & Loucks-Horsley, 1990.

mathematics activities and books. They enroll in master's program courses. These dominant modes of professional development form a substantial infrastructure readily amenable to the current press to reach large numbers of teachers, to "scale up" professional development of teachers. And yet many educators scorn and dismiss them. Do we really know what we need to know about their hidden possibilities? What are their seeds of promise? And what don't we know because it has not been tried and discussed?

The kinds of common activities named in the previous paragraph are disdained for several reasons. Some would argue that they are too brief, too weak, too fragmented. Others would point to their tendency to oversimplify: Providing teachers with activities is unlikely to help them delve into the deeper issues of changing the way they teach mathematics. Still others would note the prevailing tendency toward instrumental goals, and toward technical knowledge imported for teachers' use in their classrooms.[38] Yet these criticisms seem to focus on the structure of professional development, rather than on its conceptual orientation, content, or pedagogy.[39] A long-term teacher development project might develop enthusiasm and yet spur little in the way of serious engagement with reform. A rural Alaska teacher thoughtfully using the new *Investigations*[40] mathematics materials with her students miles from any opportunity for professional development might be more stimulated and supported.

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[38]Lord, 1994.

[39]Feiman-Nemser (1990) argues that structures alone cannot be the determining characteristic of teacher preparation programs, that consideration of conceptual orientations to what teachers need to learn, and in what ways, is crucial in designing alternative approaches.

[40]Russell & Rubin, 1994.

Structures offer us a multitude of vehicles and sites for creating possibilities for teacher learning. Some are particularly well-positioned to help meet the challenge of "scaling up" to engage many more teachers in working toward mathematics reform. We should neither write off particular structures without closer consideration, nor uncritically embrace others. For instance, how might electronic mail be a support for professional community?[41] What can people learn from videotape? Are methods courses necessarily of little use? Can mathematics curriculum materials be designed so as to be educative for teachers? How can "follow-up" be provided? That is, what would it take for any particular design to function as an opportunity for sustained learning?[42] Somehow we need to turn our attention toward the aims and orientation of teacher learning opportunities.

Exploring "stance" in the orientation of professional development

Traditionally, professional development (such as inservice workshops) and professional forums (such as journals and state meetings) assume a *stance* toward practice that concentrates on answers: conveying information, providing ideas, training in skills.[43] With enthusiasm and clever quips, leaders distribute ideas, tips, and guidance. Handouts and reproducible

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[41]Although I do not take this up here, there are examples of this worth examining more closely. See, for example, the November 1994 issue of the *Mathematics Teacher*, the special pullout section of *Education Week* (January 11, 1995), and Glazer, 1994.

[42]An image to challenge our assumptions that a deliberative discourse among teachers can only happen when teachers enjoy support, time, and autonomy is that of elementary teachers in China, where external curriculum policies are much stronger and more controlling. Paine and Ma (1993) describe how the common structure provided by a mandated curriculum supports a kind of professional discourse rarely seen in the U.S. except in the context of intensive teacher development programs. Teachers compare notes about particular lessons and problems, discuss how their students respond to specific tasks, and discuss plans.

[43]Little, 1993; Lord, 1994; Sparks & Loucks-Horsley, 1990.

worksheets are eagerly collected and filed. In some sessions, participants share ideas -- but this is still very much a discourse of answers, a confident stance of certainty. On one hand, this offers participants an enormous assortment of potential resources. However, their potential is restricted by the lack of critical discussion. Seeking to make participants comfortable, staff development leaders rarely challenge teachers' assumptions or provoke disequilibrium or conflict intentionally.[44] Because discussions of teaching sometimes resemble "style shows" more than they do professional interaction, teachers' development of their practice is often a highly individual and idiosyncratic matter. The common view that "each teacher has to find his or her own style" is a direct result of working within a discourse of practice that maintains the individualism and isolation of teaching.[45] This individualism not only makes it difficult to develop any sense of common standards, it also makes it difficult to *disagree*. Masking disagreements hides the individual struggles to practice wisely, and so removes an opportunity for learning. Politely refraining from critique and challenge, teachers have no forum for debating and improving their understandings. To the extent that teaching remains a smorgasbord of alternatives with no real sense of community, there is no basis for

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[44]Lord, 1994.

[45]I would like to acknowledge Dan Chazan for helping me see this underside of the individualistic culture of teaching. Brian Lord (1994) makes a similar argument related to individual teachers' learning. He argues that because most teachers' conceptions of knowledge, learning, and their role are fundamentally at odds with those that underlie the reform movement, individual and collective challenge and conflict are essential to integrating new ways of thinking about teaching.

comparing or choosing from among alternatives, no basis for real and helpful debate.[46] This lack impedes the capacity to grow.

With goals that are uncertain and underdetermined, a stance of certainty and of answers is unlikely to press deeply into the work of reform. We would do well to consider and experiment with fostering a stance of critique and inquiry rather than one of answers -- a stance of asking and debating, a discourse of conjecture and deliberation.[47] With norms and patterns for discussing alternatives, for arguing about relative merits, for adaptation and evaluation, many more "opportunities" could truly have the possibility of being educative.

What might characterize a stance of critique and inquiry toward practice? One aspect might be the nature of encounters with new ideas -- an important part of learning. Such a stance would strive to *make a new idea viable*, getting it on the table for examination, trial, and debate, but not pushing it as "the way." It would involve convincing others that an idea is worth considering, but without "selling" it. A second aspect might center on *considering how other resources and knowledge might be useful* in connection with particular agendas -- not as authoritative truth, but as tools for local deliberations. Examining research both inquisitively and skeptically, teachers with such a stance would seek insights from scholarship, but not accord undue truth to its conclusions. This stance would accommodate "the possibility that the available research knowledge is incomplete and there is room for discovery. [It would] neither

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[46]Robert Floden, drawing on Campbell (1974/1988), suggests that the lack of debate creates a vacuum around the need for a critical winnowing of the plethora of pedagogical ideas and practices.

[47]E.g., Fullan, 1982; Little, 1982, 1993; McLaughlin & Marsh, 1978. Lord (1994) writes about this as "critical colleagueship."

romanticize teachers' knowledge nor unduly privilege researchers' claims."[48] A third aspect might entail shifting the emphasis from "implementation" of programs to the *adaptation of innovation* and *generation of new knowledge*. Acknowledging the uncertainties and underdeterminedness of the reform visions, local interpretation and invention is inevitable.[49] And desirable. The particularities of local circumstances require tailored innovation. This stance would acknowledge this and embrace it, using the underdeterminedness of the reforms as a resource for developing inspired but locally-tailored innovations.[50]

These three aspects all deal in one way or another with relationships with new ideas -- how one might engage them, where one might seek them, and how one might develop them -- and all with a combined openness to the insights and images of others, and an awareness of the role of critique and adaptation. Missing in these is an explicit concern for community -- the final aspect which I will explore.[51] Successful teacher development projects often count among their essential elements the construction of such a community within the project.[52] Are there other ways to foster communities of practice, both real (face-to-face) and virtual? A stance of inquiry would also require a sense of membership in some *wider community* of others engaged in reform -- in seeking, hearing, envisioning, experimenting, examining, and revising.

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[48]Little, 1993, p. 143.

[49]McLaughlin, 1976.

[50]Tyack & Tobin, 1994.

[51]I assume that developing a new stance toward practice will itself entail cycles of invention, experimentation, and re-articulation.

[52]E.g., Brown, 1994; Featherstone, Pfeiffer, & Smith, 1993; Featherstone, Pfeiffer, Smith, Beasley, Corbin, Derksen, Pasek, Shank, & Shears, 1993; Simon & Schifter, 1991.

Connections with others can extend local resources. Such connections are also an antidote for the risks of the self-reflexive tendency inherent in the current enthusiasm for school-based restructuring.[53] What might be ways to create both local community and connections with a broader community, fostering access to and opportunities to distribute new knowledge and hypotheses for practice?

We need to develop and experiment with such stances within both traditional and nontraditional structures for professional development -- in the articles we write, the presentations we give, in the work we do with teachers in schools.[54] What do we know -- and *not* know -- that can inspire and support experiments with alternative stances within the material, content, and discourse of professional education?[55]

In this last section, below, I propose three sites for the refinement and testing of a set of working hypotheses about ways to engage teachers in working toward the reforms: curriculum materials, videotape, and teacher writing. The first one is conventional -- part of the

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[53]Lord (1994) asks, "In short, how can local efforts to develop critical colleagueship avoid parochialism...?" (p. 197).

[54]A special challenge for teacher educators is how unfamiliar all this is for them, too. Many are themselves developing their own practice in the spirit of the reforms, to involve and honor their learners' ideas and ways of thinking, to construct meaningful problems and tasks, and to change the discourse in which they and learners engage. They also vary in their experience with and knowledge of "this kind of teaching" in K - 12 classrooms, the albeit incomplete and uncertain "content" of that which they seek to help teachers learn.

[55]The notion that "experiments" could serve to ground and intermingle the development of both theory and practice has its origins in Dewey. Shulman (1994) argues that naturalistic experiments that situate inquiry in the mess of real world contexts are useful to the development of theory.. He advocates for "design experiments" [Brown (1992)]: hypotheses formulated as plans, adaptively carried out in a real context, and documented across the course of their evolution. This conjectural stance toward practice fits well with the inherent uncertainties. Based on evolving understandings of learners, content, and context, a disciplined approach to developing reform is to design and try out smart hypotheses and to study closely what happens in practice.

infrastructure I described above -- and the second two are novel: vehicles that offer promise and might be worth attention, care, and experimentation. Each of the three offers a structure potentially amenable to large-scale work. Each offers resources; how each is engaged by teachers could serve to extend the resources of the individual through connections with others around it. Each offers ideas for teaching; each contains the possibility for supporting the generation of new knowledge for teaching. Each holds the possibility for encouraging and supporting a stance of inquiry and experimentation, of critique and deliberation. I propose these as examples of working hypotheses that we will need to develop, refine, test, revise, and try again, if we are to meet the challenges of supporting teachers' learning.

Working hypotheses: Scaling up with a stance of inquiry

The following three sketches are illustrative. None is yet the design for an experiment.[56] Each involves resources of practice -- images, understandings, ideas, ways of being or deliberating -- and each involves an effort to develop a pedagogical stance that fosters inquiry and critique.

Using redesigned curriculum materials

Influenced by a big backlash against the teacher-proof curriculum movement, contemporary educators often disparage textbooks, and many reform-oriented teachers -- emissaries of the reforms -- repudiate them, announcing disdainfully that they do not use textbooks. Yet carefully designed curriculum materials could offer teachers access to mathematical ideas and ways to represent them. Curriculum materials could serve as a rich

[56]Brown, 1992.

site for ongoing teacher learning.[57] They could offer maps of the mathematical territory, helping teachers to reconceive that terrain in ways that reconfigure it around "big ideas."[58] They could provide alternative tasks and discuss their relative advantages and pitfalls. They could offer teachers forecasts of students' likely thinking. With a stance of contributing to an ongoing effort to teach, to a conversation about possibility, text materials would seem to hold untapped potential.

Curriculum could be written with teacher learning as a goal.[59] Most curriculum developers have their eye on students rather than on teachers, and attempt to guide teachers without engaging them in pedagogical conversation.[60] To what extent do textbook authors aim to help teachers learn mathematics through the materials they write? And what would it take for teachers to engage in readings -- and uses -- of such texts that would not convert them to their traditional position of external authoritative guide for the activities of teachers and students?

As teachers build their own understandings and relationships with mathematics, they chart new mathematical courses with their students. And conversely: As they move on new paths together with students, their own mathematical understandings change. Whether and

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[57]Remillard, in preparation.

[58]E.g., Lappan, Fey, Fitzgerald, Friel, & Phillips, in press; Russell, Schifter, Bastable, Yaffee, Lester, & Cohen, 1994.

[59]Russell, 1994.

[60]Remillard (in preparation) writes about a vision of textbooks that speak *to* rather than *through* teachers, and explores how teachers might come to hold such an expectation for their relationship with a text.

how curriculum materials can be designed to support teachers' exploration of mathematics -- their own and their students' -- is a question worth fresh investigation. Given the expanse of mathematics to be learned, and the multiple ways in which it can be explored, it would be worthwhile to investigate whether and how materials designed to support both teachers' and students' learning could function as resources for teacher learning rather than as controls for teachers' coverage. Furthermore, there are sites already in place for such exploration. For example, several contemporary professional development projects use curricula as the stimuli for conversations among teachers about teaching.[61] Using the texts in their own classrooms, reporting on what happened, reflecting on the strengths and weaknesses of different ideas and activities, the teachers in these projects learn about teaching and learning, mathematics and reform.

These projects resurface the crucial pedagogical issue: While curricula could be designed with teacher learning in mind, what teachers learn from such materials will also depend on the ways in which they are engaged with them, what the norms and expectations are surrounding their use. What might be the time frame over which a teacher develops a relationship with the curriculum material, and how might the third year of use be different from the first? Perhaps texts might be deliberately designed to be "outgrown." We have much to learn about the pedagogy of using such materials to support and facilitate teacher learning. In what ways could experiences be shaped around these materials in order to enhance their educative potential?

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[61]E.g., the Renaissance Project in California, directed by Judith Mumme (see Acquarelli & Mumme, in press).

But these are issues worth working on. Textbooks continue to be a mainstay of the elementary classroom in most schools, and as such, find their way to teachers' elbows and into the daily ticking of their practice. Designing ways to use them more directly in the service of teacher development is strategic.

Watching videotape from classrooms where teachers are seriously engaged in efforts to teach mathematics differently

The need for "images of reform" is widely-touted. Teachers who have never seen children engaged in a mathematics problem, or discussing mathematics, need to have opportunities to see what this can look like. These serve, in part, as existence proofs that such practice can happen in schools.[62]

My experience with watching educators (and others) watch, talk about, and refer to videotapes of classroom lessons suggests that, despite the widespread enthusiasm for the medium, we know little about what people attend to and learn while watching tape.[63] Do these tapes infuse new images alongside the deeply-ingrained ones from more conventional classrooms? If so, what aspects of these images are salient -- the kind of mathematics, the nature of the discourse, the capabilities of students, the teacher's role? All of these? Perhaps some viewers study teacher moves, voice, stance -- and deliberately or unconsciously "try on" unfamiliar ways of being with students. I have seen teachers experiment with asking questions

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[62]This can easily backfire. Teachers can simply dismiss what they see: "These kids are just very bright -- my students would not be able to do this." "This cannot happen every day."

[63]I draw here on my work with Magdalene Lampert, Kara Suzuka, Ruth Heaton, Angie Eshelman, and Mark Rosenberg, in which we have been investigating the use of primary source materials from Lampert's and my fifth and third grade mathematics classes in teacher education contexts (e.g., Lampert & Ball, 1990; Ball, Lampert, & rosenberg, 1991).

like a teacher on tape, and then note the interesting differences in how their students respond. Such imitation, usually disparaged as not "educative," is something we know little about. Perhaps there are things having to do with ways of being with students, ways of being in oneself, that can be supported through the viewing of tape.[64] Perhaps there are subtle aspects of interaction and manner that are not available for comment or examination in written accounts of teaching, in curriculum materials, or in other kinds of professional development opportunities. What can be learned from videotapes, under what kinds of circumstances, is worth investigating much more closely.

An associated question involves the kinds of tapes and teaching used. What is offered by polished professional quality tapes? What do rough, problematic cases afford? When is watching a novice teacher preferable and why? When are the struggles of experienced teachers crucial to see? Annotations layered onto the video can shape the viewing; we know little about how they affect viewers' opportunities. What features of who the teacher is seem to affect viewers' reactions? Lampert and Eshelman (1995) write about the development of prospective teachers' capacity to view teaching with empathy. Being able to imagine yourself as the teacher on the tape, or being able to understand the teacher's perceptions and decisions whether or not you agree -- these seem important for a viewing that connects the viewer with

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[64]Common interpretations of constructivist theory leaves little room for imitation as a potent form of learning some kinds of things. Yet, in learning to play the piano, listening to a skilled and talented pianist can help develop one's ear, to get a feel for interpretation, to acquire position, motion, and timing (for a marvelous account of what this might be like, see Frank Conroy's novel *Body and Soul*). It is quite likely that there are unexamined aspects of teaching that bear a strong relationship to some of these difficult-to-capture aspects of piano playing.

the tape, and thus for the learning that it can make possible: to see through that teacher's eyes, to feel what she feels, to think with her heart and mind.

Necessarily, a videotape is but one slice of classroom life. We know little about the most helpful "slices": Should tapes focus on children and their talk? Should tapes highlight the teacher and her moves? Are some aspects of the curriculum more important to document in such tapes? Maybe an old tape will do, but I doubt it. What is afforded by the availability of additional material, such as copies of children's work, teacher reflections, assessment items? Considering the different aspects or features of tapes that might be significant and exploring the range of their impacts is an important part of learning how these tapes might (or might not) be helpful.

Another, equally important question involves the "pedagogy" of using videotapes. Like any materials, what people learn from the tapes is influenced both by what they bring (e.g., assumptions and values, experiences and beliefs) and how they are engaged (some might say, taught) while viewing the tape. What kinds of discussions are most fruitful? Are there alternative organizational structures in which to use tapes (small groups versus large group settings, pre-viewing, and structured observation during the viewing all come to mind as possibilities)? Are there ways to direct -- or widen -- participants' attention so as to take the most advantage of the viewed tape? And perhaps, most thorny of all, is the challenge of developing a stance that is less simply evaluative and more analytic of practice.

The small but growing body of literature within teacher education about the use of cases would be a useful source of insight to questions such as these. Scholars and teacher educators

have begun to consider the possibilities of using cases to focus and ground discussions.[65] While much discussion focuses on the question, "What constitutes a good case?" other discussion focuses on the question, "How does one teach a case?" This is an equally significant question for the viewing of videotapes: How does one structure the experience of viewing in ways that generate learning?

Videotapes have great potential for "reach." Easily distributable to large numbers of people, tapes hold promise as a site for learning. But we need to probe better what shapes fruitful uses, as well as how guidance can be provided for a variety of kinds of use. When, for example, might it make sense to use a tape to *exemplify* a kind of teaching and learning? Under what circumstances might it make sense to use videotape as a springboard for investigation -- of the particulars of that tape, as well as of more general issues of teaching, learning, mathematics, purposes?

Reading and writing a literature about efforts to work toward reform in mathematics teaching

A third possible site is writing (and, hence, reading) about practice. The writing process community has, quite appropriately, spawned a literature produced by writing teachers. These accounts provide glimpses of teachers' work behind their classroom doors.[66] Other teachers find within these stories both inspiration and solace. Practical tips and ideas can be found as well. Many elementary teachers devour these titles (much like a novel by one's favorite

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[65]Sykes & Bird, 1992; Merseth, 1991. See the work of both Lee and Judy Shulman in this area (e.g., L. Shulman, 1992; J. Shulman, 1992).

[66]E.g., Elbow, 1986; Graves, 1983; Routman, 1988, 1991.

author), and await the publication of new ones, or the next installment. They seem to fill a void in the discourse about teaching -- in this case, the teaching of writing.

The beginnings of such a literature are emerging in mathematics education.[67] Will teachers read such accounts with anything like their appetite for the parallel volumes in writing? It is a genre of professional literature that is underdeveloped and underexplored, but worth investigating. Some teacher development projects have engaged teachers in writing about their efforts as a tool of professional development and found it to be a powerful vehicle for conversation about teaching and learning, both with oneself (the author) and others.[68] Still, we are only beginning to explore what it might take to help teachers write such books, and differences in the qualities of such texts: What features -- content, tone, narrative quality -- might affect what teachers gain or learn from reading them? For example, how are upbeat, positive accounts read as compared with ones that reveal struggles, tensions, and uncertainties? How are different authorial stances represented in the pages of these books, and how do they affect readers' experiences of them? Can such books offer teachers paths into other literatures, and would they pursue them, under what conditions? Some of the writing books are accompanied by annotated bibliographies. How are these used, and what impact does further reading have?

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[67]E.g., Burns & Tank, 1988; Featherstone & Beasley, in preparation; Heaton, 1994; Romagnano, 1994; Schifter, 1994, in press a and b.

[68]Barnett, 1991; Schifter, in press; Shulman & Colbert, 1988.

We know still less about how teachers who would buy and read books about teaching and learning mathematics might use them. How might such books be promoted and distributed? In what contexts might teachers read them (e.g., alone, as part of a Book Club, as a building staff) and how might these shape the books' role in teachers' practice? The interesting success of this burgeoning literature in the writing movement should encourage those working for mathematics reform to consider and explore the potential.

Still, differences exist between language arts and mathematics that should not be ignored. A key aspect of the writing process work is that it offers a way for teachers to learn in the company of other teachers, and to use one another as resources in learning writing, as well as in teaching writing. The medium *is* the content. Teachers can come to see themselves as writers as they write about their teaching of writing. Teachers' own resources in mathematics tend to be thinner than in reading and writing. Would their writing about mathematics teaching and learning offer similar benefits? Would the products of their writing be as substantial and rich for others teachers' learning?

Associated with this are questions about the writing itself. The aims of the writing projects to which I referred above sit exploratively on a line between a strategy for supporting teacher reflection and inquiry, and the desire to develop a written discourse about teaching that might be shared. On one hand, the potential of private writing -- a teaching journal, for example -- is an underexplored medium for learning. On the other, the lack of opportunity for connections, for professional exchange, is also well-known. Might writing and sharing writing

help to foster new forms of professional communication, community -- and learning? And what does it take to do that?

What kinds of learning might be fostered by writing about one's practice? Related to stance are questions of tone and focus, purpose and audience. What is gained from writing "confessional" pieces of one's struggles? Or in the voice of one who has made a Great Change, with a great new world of rosy answers? What about analytic pieces that grapple with some recurring dilemma, a challenge, a student? What about reflections on self?

There is the recurring -- and essential -- question of pedagogy: What does one need to do to support and facilitate such writing? What risks are associated with the revelations and sharing they may entail? What might be entailed in supporting the sharing of texts about teaching? How can sharing and response develop, and what are some of the pitfalls?

CONCLUSION:

WHAT MIGHT IT MEAN TO "SCALE UP" PROFESSIONAL DEVELOPMENT TO SUPPORT MORE TEACHERS' LEARNING?

In the face of the pressure to increase the "reach" of federally-funded professional development, one approach is to identify those things we believe "work" in teacher development, and literally "scale up" their size. Figure 1 provides an image of this approach.

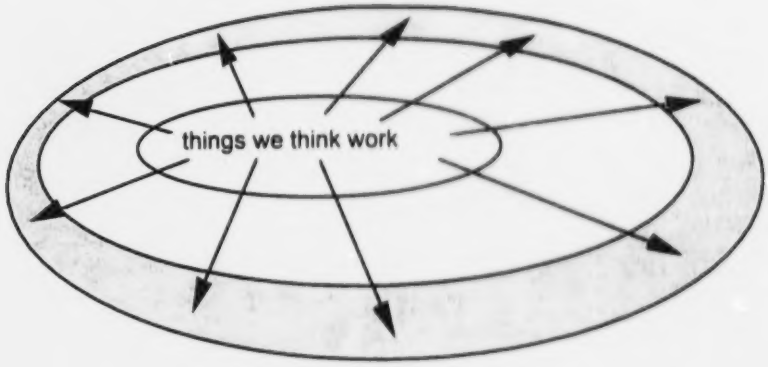

Figure 1. Expanding the "reach" of professional development: Scaling up from "things" we think work

For example, one might take the model of the small teacher study group and extend it by having each participating teacher in such a study group run her own additional teacher group. One might take the long-term focused teacher development project -- school-based, and with a weekly mathematics seminar -- and institute these in tens of locations. No doubt there is merit in trying to extend the work that has already often successfully supported teachers' growth. But "reaching" hundreds of times as many teachers seems unlikely through this approach alone. This approach tends to underestimate the role of the leader of such groups: what the qualifications and resources for the role are, and what it might take to learn to do it that is different from working on one's own teaching. Moreover, the successful teacher development projects are intensive, personal, resource-dependent, and do not always lend themselves to direct "scaling up."

Therefore, a second important tack is to consider the core conceptual elements of successful professional development projects. For example, many of the successful projects

involve long-term follow-up support for participating teachers. Are there other ways to understand both what is fundamentally important about this kind of support, and make conjectures about how else such follow-up could be provided? Conceptual, rather than model-oriented, scaling up holds more promise for dramatically increasing American teachers' opportunities for good professional development. This paper offers a start on such "conceptual" scaling up, with its effort to identify what we think we know about teacher learning. Worth remembering, however, is that our evidence for those beliefs is uneven, and we still have work to do to understand what is important, as well as limited, about various items on that list. We also have more to learn about aspects of teacher learning that the list omits. We know little, for example, about how teachers develop the personal qualities important for more complex forms of teaching -- qualities such as courage, confidence, and curiosity. We understand little about how some teachers develop a dissatisfaction with their current practice and the desire and imagination to experiment and study the results of those experiments. We know little about how teachers learn to teach sensitively and well students who are different from themselves.

A third tack is to consider what we *don't* know about professional development and to use this demand for scaling up to experiment with approaches well-suited to larger-scale teacher involvement. Figure 2 offers an image of this approach.

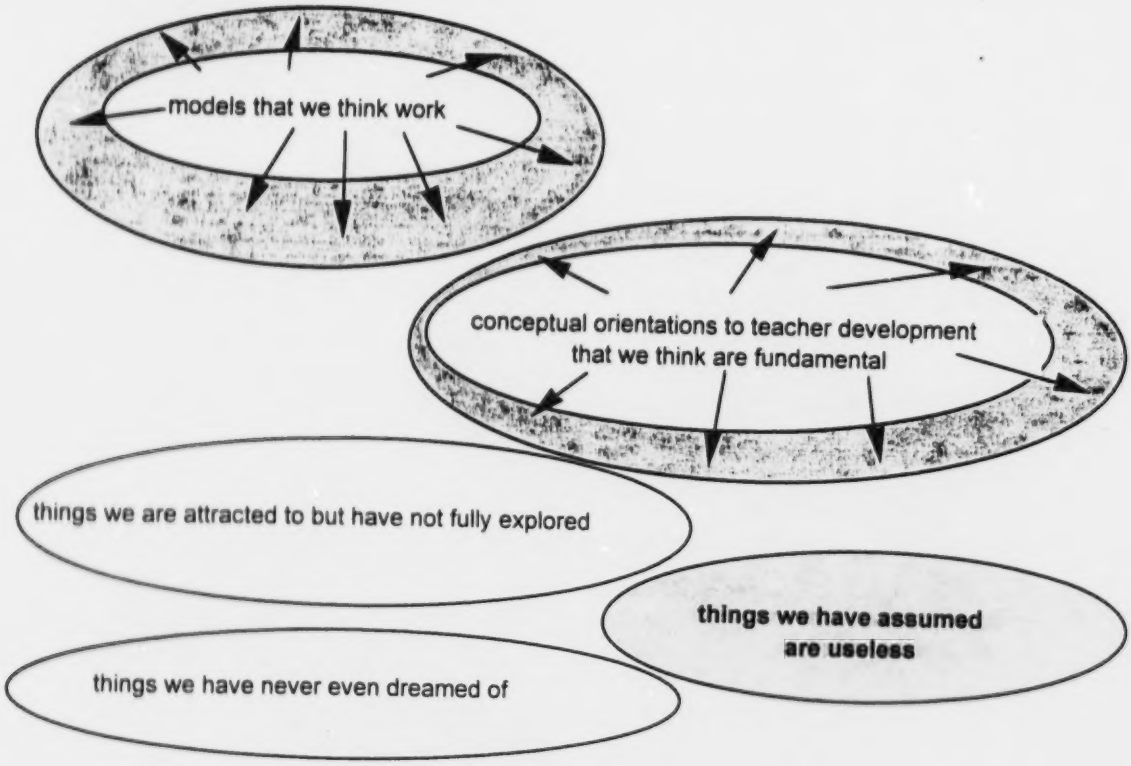
models that we think work
conceptual orientations to teacher development
that we think are fundamental
things we are attracted to but have not fully explored
things we have assumed
are useless
things we have never even dreamed of

Figure 2. Expanding the "reach" of professional development:

A mixed model of scaling up and pursuing new hypotheses

This paper also offers a beginning on this third tack. Although many have been quick to dismiss curriculum materials, I argue that textbooks and other materials are ideally situated to "reach" more teachers, and that we would do well to explore how they could be designed in ways that would especially support teacher development. And while many are enamored of the potential of videotape as a resource for professional development, I argue that we need to experiment with their potential as a tool for teacher learning.

I offer these alternatives to stimulate thought and invite participation in thinking about how to engage teachers in professional development opportunities that will support their learning and push the reforms forward. Others exist. Take the short-term workshop, the

bread-and-butter of staff development offices. Are there no things that lend themselves to this format? Are there stances one can take, pedagogical approaches one might try, within such workshops that would alter their assumed limitations? Can series of these be situated in ways that are more generative than the kinds of things we have come to expect of after-school inservice?

My stance is one of inquiry, not certainty, of questions, not answers. The work of professional development is as uncertain as practice itself.[69] The teaching we are trying to help teachers learn is underdetermined, not reducible to programs of practice. Likewise, our understanding of professional development that can support teachers' learning is a mix of myth, belief, and conjecture. Currently, we understand somewhat, but incompletely, what helps teachers learn. We understand, but need to uncover more, about what the resources are that matter in trying to teach all students well. We need to understand better the differences (and similarities) between learning to teach as a beginning teacher and changing or developing one's teaching as an experienced teacher. Adding the challenge of how to engage a much larger number of teachers in the work of these ambitious reforms makes the work all the more uncertain. As teacher educators, teachers, and policymakers, we ourselves will need to make new conjectures based on what we think we know and what we think could be. Our challenge is to experiment, study, reflect on, and reformulate our hypotheses if we are to make progress in engaging a wider community in the work of the mathematics reforms.

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[69]I am grateful to Suzanne Wilson for pointing out this notable parallel, as well as for much other wisdom about teacher learning -- her own, mine, and others'.

References

Acquarelli, K. & J. Mumme. (in press). *A renaissance in mathematics education reform.* Phi Delta Kappan.

Aichele, D. B. (Ed.) (1994). *Professional development for teachers of mathematics. Fifty-seventh Yearbook of the National Council of Teachers of Mathematics.* Reston, Va.: NCTM.

Anderson, L. M. (1989). Learners and learning. In Reynolds, M. C. (Ed.), *Knowledge base for the beginning teacher* (pp. 85-99). New York: Pergamon Press.

Ball, D. L. (1988). Unlearning to teach mathematics. *For the Learning of Mathematics, 8* (1), 40-48.

Ball, D. L. (1991). Implementing the NCTM Professional Standards for Teaching Mathematics: Improving, not standardizing, teaching. *Arithmetic Teacher, 39* (1), 18-22.

Ball, D. L. (1992). Magical hopes: Manipulatives and the reform of mathematics education. *American Educator, 16* (2), 14-18, 46-47.

Ball, D. L., M. Lampert, and M. L. Rosenberg. (1991, April). *Using hypermedia to investigate and construct knowledge about mathematics teaching and learning.* Paper presented at the annual meeting of the American Educational Research Association, Chicago, IL.

Ball, D. L. and S. Rundquist. (1992). Collaboration as a context for joining teacher learning with learning about teaching. In D. K. Cohen, M. W. McLaughlin, & J. E. Talbert (Eds.), *Teaching for understanding: Challenges for practice, research, and policy* (pp. 13-42). San Francisco: Jossey Bass.

Ball, D. L. and S. M. Wilson. (1996). Integrity in teaching: Recognizing the fusion of the moral and the intellectual. *American Educational Research Journal*, 33, 155-192..

Barnett, C. (1991). Building a case-based curriculum to enhance the pedagogical content knowledge of mathematics teachers. *Journal of Teacher Education, 42* (4), 263-272.

Borko, H., M. Eisenhart, C. A. Brown, R. G. Underhill, D. Jones, and P. C. Agard. (1992). Learning to teach hard mathematics: Do novice teachers and their instructors give up too easily? *Journal for Research in Mathematics Education, 23* (3), 194-222.

Brown, A. L. (1992). Design experiments: Theoretical and methodological challenges in creating complex interventions in classroom settings. *The Journal of the Learning Sciences 2*, 141-178.

Brown, A. L. (1994). The advancement of learning. *Educational Researcher, 23* (8), 4-12.

Brown, C. A. and H. Borko. (1992). Becoming a mathematics teacher. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 209-239). New York: Macmillan.

Burns, M. & B. Tank. (1988). *A collection of math lessons.* Salinas, CA: Math Solutions.

Campbell, D. T. (1988). Evolutionary epistemology. In S. Overman (Ed.), *Methodology and epistemology for social science: Selected papers of Donald T. Campbell.* Chicago: University of Chicago Press. (Originally published in 1974.)

References

Carpenter, T. and E. Fennema. (1992). Cognitively guided instruction: Building on the knowledge of students and teachers. In W. Secada (Ed.), *Curriculum reform: The case of mathematics in the United States. International Journal of Educational Research*, pp. 457-470. Elmsford, NY: Pergamon.

Chazan, D. and D. L. Ball. (1995). *Beyond exhortations not to tell: The teacher's role in discussion-intensive pedagogy.* (Craft Paper 95-2). East Lansing: Michigan State University, National Center for Research on Teacher Learning.

Cobb, P. (1994). Where is the mind? Constructivist and sociocultural perspectives on mathematical development. *Educational Researcher, 23* (7), 13-20.

Cohen, D. K. (1989). Teaching practice: Plus ca change.... In P. W. Jackson (Ed.), *Contributing to educational change: Perspectives on research and practice* (pp. 27-84). Berkeley CA: McCutchan.

Cohen, D. K. (in preparation). *Teaching practice and its predicaments.* Manuscript in preparation. Ann Arbor, MI: University of Michigan.

Cohen, D. K. and C. A. Barnes. (1992). Pedagogy and policy. In D. K. Cohen, M. W. McLaughlin, & J. E. Talbert (Eds.), *Teaching for understanding: Challenges for practice, research, and policy* (pp. 207-239). San Francisco: Jossey Bass.

Conroy, F. (1993). *Body and soul.* New York: Dell Publishing.

Elbow, P. (1986). *Embracing contraries: Explorations in learning and teaching.* New York: Oxford University Press.

Featherstone, H., L. Pfeiffer, S. P. Smith, K. Beasley, D. Corbin, J. Derksen, L. Pasek, C. Shank, and M. Shears. (1993, April). *"Could you say more about that?" A conversation about the development of a group's investigation of mathematics teaching.* Paper presented at the annual meeting of the American Educational Research Association, Atlanta GA.

Featherstone, H., L. Pfeiffer and S. P. Smith. (1993). *Learning in good company: Report on a pilot study* (Research Report 93-2). East Lansing: Michigan State University, National Center for Research on Teacher Learning.

Featherstone, H. and K. Beasley. (in preparation). *"The big old conversation": Reflections on mathematical tasks and discourse.*

Feiman-Nemser, S. (1990). Teacher preparation: Structural and conceptual alternatives. In W. R. Houston (Ed.), *Handbook of research on teacher education* (pp. 212-233). New York: Macmillan.

Fullan, M. G. (1982). *The meaning of educational change.* New York: Teachers College Press.

Glazer, E. (1994). *An analysis of discourse on an Internet-based listserv for mathematics educators.* Unpublished master's thesis, Champaign-Urbana: University of Illinois.

Goodlad, J. I. (1984). *A place called school: Prospects for the future.* New York: McGraw-Hill.

References

Grant, C. A. and W. G. Secada. (1990). Preparing teachers for diversity. In W. R. Houston (Ed.), *Handbook of research on teacher education* (pp. 403-422). New York: Macmillan.

Graves, Donald H. (1983). *Writing: Teachers and children at work*. Exeter, N.H.: Heinemann.

Heaton, R. M. (1994). *Creating and studying a practice of teaching elementary mathematics for understanding*. Unpublished doctoral dissertation, East Lansing, MI: Michigan State University.

Jackson, P. W. (1986). *The practice of teaching*. New York: Teachers College Press.

Lakatos, I. (1974). *Proofs and refutations: The logic of mathematical discovery*. Cambridge: Cambridge University Press.

Lampert, M. (1985). How do teachers manage to teach? Perspectives on problems in practice. *Harvard Educational Review, 55*, 178-194.

Lampert, M. and D. L. Ball. (1990). *Using hypermedia technology to support a new pedagogy of teacher education*. (Issue Paper 90-5). East Lansing: Michigan State University, National Center for Research on Teacher Education.

Lampert, M. and A. S. Eshelman. (1995, April). *Using technology to support effective and responsible teacher education: The case of interactive multimedia in mathematics methods courses*. Paper presented at the annual meeting of the American Educational Research Association, San Francisco.

Lappan, G., J. T. Fey, W. M. Fitzgerald, S. N. Friel, E. D. Phillips. (in press). *Connected Mathematics Project*. Palo Alto, CA: Dale Seymour Publications.

Little, J. W. (1982). Norms of collegiality and experimentation: Workplace conditions of school success. *American Educational Research Journal, 19* (3), 325-340.

Little, J. W. (1993). Teachers' professional development in a climate of educational reform. *Educational Evaluation and Policy Analysis, 15* (2), 129-151.

Lord, B. (1994). In N. Cobb (Ed.), *The future of education: Perspectives on national standards in America*. New York: College Entrance Examination Board.

Lortie, D. C. (1975). *Schoolteacher: A sociological study*. Chicago: The University of Chicago Press.

McLaughlin, M. W. (1976). Implementation as mutual adaptation. *Teachers College Record, 77*, 339-351.

McLaughlin, M. W. and Marsh, D. D. (1978). Staff development and school change. *Teachers College Record, 80* (1), 69-94.

Merseth, K. (1991). *The case for cases*. Washington DC: American Association for Higher Education.

Mirel, J. (1994). School reform unplugged: The Bensenville New American School project, 1991-1993. *American Educational Research Journal, 31* (3), 481-518.

References

National Council of Teachers of Mathematics, (1989). *Curriculum and evaluation standards for school mathematics*. Reston, Va.: Author.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, Va.: Author.

National Research Council. (1991). *Everybody counts: A report to the nation on the future of mathematics education*. Washington, D.C.: National Academy Press.

National Research Council. (1993a). *Measuring up: Prototypes for mathematics assessment*. Washington, D.C.: National Academy Press.

National Research Council. (1993b). *Measuring what counts: A conceptual guide for mathematics assessment*. Washington, D.C.: National Academy Press.

Paine, L. and L. Ma. (1993). Teachers working together: A dialogue on organizational and cultural perspectives of Chinese teachers. *International Journal of Educational Research, 19*, 675-697.

Remillard, J. (in preparation). *Changing texts, teachers, and teaching: The role of textbooks in reform in mathematics education*. Unpublished doctoral dissertation, East Lansing, MI: Michigan State University.

Romagnano, L. (1994). *Wrestling with change: The dilemmas of teaching real mathematics*. Portsmouth, NH: Heinemann.

Roosevelt, D. (1994). *Constructing a self: Studying trust, respect, and responsiveness in teaching*. Dissertation in progress.

Routman, R. (1988). *Transitions: From literature to literacy* (1st edition). Portsmouth, NH: Heinemann.

Routman, R. (1991). *Invitations: Changing as teachers and learners K-12*. Portsmouth, NH: Heinemann.

Russell, S. J. (1994). *The role of the teacher in curriculum development (OR, Won't well-prepared teachers make up their own?) (OR, Curriculum: The Right way, a necessary evil, a handy reference, or partner?)* Paper prepared for the conference on teacher enhancement K-6, National Science Foundation, Arlington, VA.

Russell, S. J. and A. Rubin, (1994). *Landmarks in the hundreds*. In *Investigations in Number, Data, and Space*. Palo Alto, CA: Dale Seymour Publications.

Russell, S. J., D. Schifter, V. Bastable, L. Yaffee, J. Lester, S. Cohen. (1994, November). *Learning mathematics while teaching*. Paper presented at the annual meeting of the North American Chapter of the Psychology of Mathematics Education, Baton Rouge, LA.

Schifter, D. (1993). *Reconstructing mathematics education: Stories of teachers meeting the challenge of reform*. New York: Teachers College Press.

Schifter, D. (1994). *Voicing the new pedagogy: Teachers write about learning and teaching mathematics*. Center for the Development of Teaching Paper Series. Newton, MA: Education Development Center.

References

Schifter, D. (in press). *Constructing new practices/Reconstructing professional identities: Teacher narratives from the mathematics education reform movement.* New York: Teachers College Press.

Schifter, D. (in press). Voicing the new pedagogy: Teachers interpret the rhetoric of mathematics education reform. New York: Teachers College Press.

Shulman, J. H. (Ed.) (1992). *Case methods in teacher education.* New York: Teachers College Press.

Shulman, L. S. (1983). Autonomy and obligation: The remote control of teaching. In L. Shulman & G. Sykes (Eds.), *Handbook of teaching and policy*, (pp. 484-504). New York: Longman.

Shulman, L. S. (1992). Toward a pedagogy of cases. In J. H. Shulman (Ed.), *Case methods in teacher education* (pp. 1-30). New York: Teachers College Press.

Shulman, L. S. (1994, April). *From Brownell to Ball.* Invited address at the American Educational Research Association, New Orleans, LA.

Shulman, J. H. and J. A. Colbert. (1988). *The intern teacher casebook.* San Francisco: Far West Laboratory for Educational Research and Development.

Simon, M. A. (1993). Prospective elementary teachers' knowledge of division. *Journal of Research in Mathematics Education, 24* (3), 233-254.

Simon, M. A. and D. Schifter. (1991) . Towards a constructivist perspective: An intervention study of mathematics teacher development. *Educational Studies in Mathematics, 22*, 309-331.

Smith, J. (in press). Efficacy and teaching mathematics by telling: A challenge for reform. *Journal for Research in Mathematics Education.*

Sparks, D. and S. Loucks-Horsley. (1990). Models of staff development. In W. R. Houston (Ed.), *Handbook of research on teacher education* (pp. 234-250). New York: Macmillan.

Sykes, G. and T. Bird. (1992). Teacher education and the case idea. In G. Grant (Ed.), *Review of Research in Education, 18* (pp. 457-521). Washington, DC: American Educational Research Association.

Theule-Lubienski, S. (in preparation). *Mathematics for all? A closer look at teaching and learning mathematics in class.* Unpublished doctoral dissertation, East Lansing, MI: Michigan State University.

Toney, N. (in press). Facing racism in mathematics education. In D. Schifter (Ed.), *Constructing new practices/Reconstructing professional identities: Teacher narratives from the mathematics education reform movement.* New York: Teachers College Press.

Tyack, D., and W. Tobin. (1994). The "grammar" of schooling: Why has it been so hard to change? *American Educational Research Journal, 31* (3), 453-479.

Weissglass, J. (1994). Changing mathematics teaching means changing ourselves: Implications for professional development. In Aichele, D. B., (Ed.), *Professional development for teachers of mathematics. Fifty-seventh yearbook of the National Council of Teachers of Mathematics* (pp. 67-78). Reston, VA: NCTM.

References

Wilson, S. M., L. S. Shulman and A. E. Richert. (1987). "150 different ways" of knowing: Representations of knowledge in teaching. In J. Calderhead (Ed.), *Exploring teachers' thinking*. London: Cassell.